DEFINITIONS AND THEORY

1. Static equilibrium:

We define static equilibrium as the necessary and a sufficient condition for a non-deformable body to be in mechanical equilibrium and therefore the net force acting upon the particle is zero. This condition implicates the next:

$$\sum F_i = 0 \qquad \sum M_i = 0$$

This is, six scalar equations:

$$\sum F_x = 0; \sum F_y = 0; \sum F_z = 0$$

$$\sum M_x = 0; \sum M_y = 0; \sum M_z = 0$$

This condition is not sufficient for deformable a body which also needs to accomplish the equations o internal equilibrium.

2. Stability

Before calculating reactions forces in structures it is necessary to determine if this calculation is possible or not. In order to do this the next process will be followed:

1. Determine the EDSI in case there are no internal links.
2. In case of having a structure with internal links, besides we will calculate the internal degree of static.
3. Determine the global degree of static indeterminacy. In case of a DSI greater than zero:
4. Check by inspection the structure. This is to check there is nothing "strange" in the structure such as if all reactions are concurrent at a point or if the bars of a truss are connected in a way that it is not possible to withstand every system of forces. In order words, before stating the character of the structure, it should be verified that there is not any pattern of displacements on the system

External Degree of static indeterminacy

It is expressed like this:

$$\mathbf{EDSI = R - EDOF}$$

Where each term is:

- EDSI: external degree of static indeterminacy
- R: number of reaction forces
- EDOF: external degrees of freedom (for example a point in a 3d space is three and solid rigid body restrain by a line has two: a rotation and a translation).

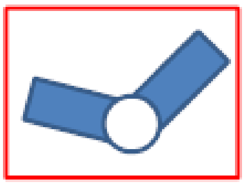
In function of the prior results it can be stated:

- If EDSI < 0 then the structure is externally a mechanism
- Then if EDSI = 0 it means the structure is externally statically determined
- If EDSI > 0 then the structure is externally statically undetermined

Internal Degree of static indeterminacy

There are two types of internal links.

a) The first one consists in bars jointed by a hinge joint like the next one shown in the figure



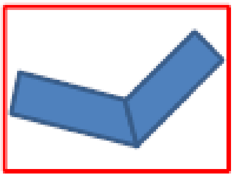
In this configuration it is possible to quantity the number of internal links by:

$$\mathbf{IL = 2.(b-1)}$$

Where each term is:

- IL: Internal links
- b: number of bars jointed to a joint

b) The second one consists in bars jointed by a rigid joint



In this case, the internal links are determined by:

$$\mathbf{IL = 3.(m-1)}$$

Where each term is:

- IL: Internal links
- m: number of bars jointed to a frame joint

Anyway, the number of internal degrees of freedom is calculated using the next expression:

$$\mathbf{IDOF = 3.(m-1)}$$

Where each term is:

- IDOF: internal degrees of freedom
- m: number of bars jointed to a frame joint

Finally, the internal degree of static indeterminacy (IDSI) is defined as:

$$\mathbf{IDSI = IL - IDOF}$$

Now there are three possible scenarios:

- If IDSI<0 then the structure is internally a mechanism
- Then if IDSI= 0 it means the structure is internally statically determined
- If IDSI>0 then the structure is internally statically undetermined.

Global Degree of static indeterminacy (DSI)

The DSI is defined as:

$$\mathbf{DSI = EDSI + IDSI}$$

Now there are three scenarios:

- If DSI<0 then the structure is globally a mechanism
- Then if IDSI= 0 it means the structure is globally statically determined
- If IDSI>0 then the structure is globally statically undetermined.

Direct calculation of the DSI without evaluating EDSI or IDSI

It is possible to obtain immediately the DSI of a structure by applying the next equation:

$$\mathbf{DSI = r + m - 2j}$$

Where each term is:

- R: number of reaction forces
- m: number of members
- j: number of hinge joints.

In case of a structure exclusively design with frame joints then the equation is:

$$\mathbf{DSI = r + 3m - 3j}$$

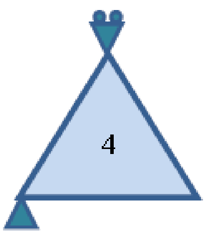
Where j:

- j: number of rigid joints

SOLVED EXERCISES

Calculate for the next structures: its external, internal and global degree of static indeterminacy and explain its implications.

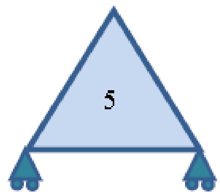
Triangular plate (I)



$\mathrm{EDSI = R - EDOF = 3 - 3 = 0}$

Furthermore, its **reaction forces are not concurrent in a point**, then the system is **entirely linked**, which means its reactions forces can support any external force system and they can be determined because its EDSI is equal to zero.

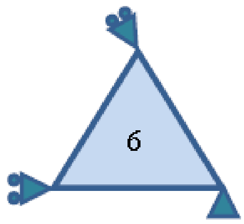
Triangular plate (II)



$\mathrm{EDSI = R - EDOF = 2 - 3 = -1}$

Thus, the system is partially linked which means that the reactions can only support some external force systems. It is also called **mechanism.**

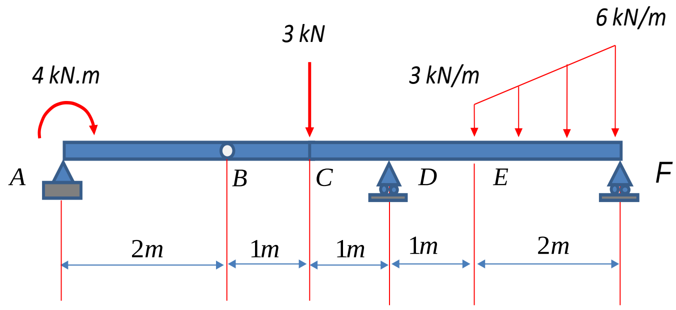
Triangular plate (III)



$\mathrm{EDSI = R - EDOF = 4 - 3 = 1}$

However, all reaction forces are concurrent in the bottom right corner, and as a result the structure is **partially linked or mechanism.**

Beam (I)



$\text{EDSI} = 4 - 3 = 1$

$\text{IDOF} = 3(2 - 1) = 3$

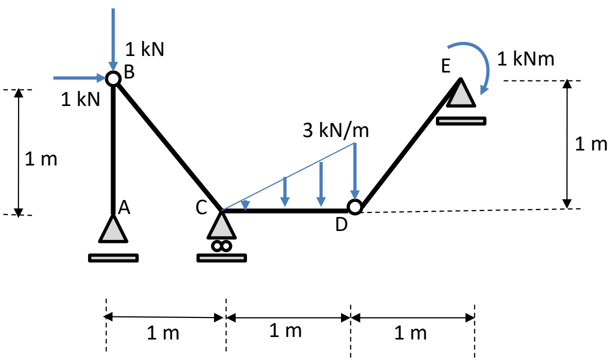
$\text{IL} = 2(2 - 1) = 2$

$\text{IDSI} = 2 - 3 = -1$

$\text{DSI} = \text{EDSI} + \text{IDSI} = 1 - 1 = 0$

The structure is completely linked and the reactions are statically determined.

Beam (II)



$\text{EDSI} = 5 - 3 = 2$

$\text{IDOF} = 3(4 - 1) = 9$

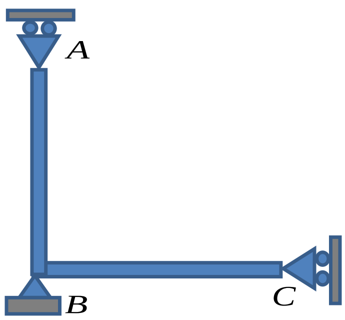
$\text{IL} = 2[2(2 - 1)] + 3(2 - 1) = 7$

$\text{IDSI} = 7 - 9 = -2$

$\text{DSI} = \text{EDSI} + \text{IDSI} = 2 - 2 = 0$

The structure is completely linked and the reactions are statically determined.

Frame (I)



There are no internal links, just external, and therefore since we have four reactions forces (two of the pin support and another two due to the couple of simply support) and three degrees of freedom (the beam can move upwards, downwards and can rotate about the perpendicular axis) the value of the EDSI is:

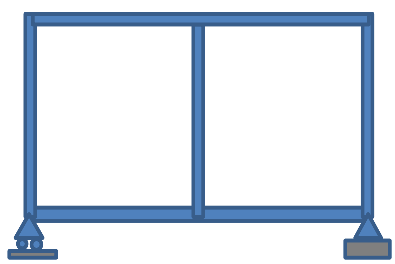
$\text{EDSI} = 4 - 3 = 1$

This means that the structure is statically indeterminate and it would not be possible to determine reaction forces (4 unknown factors and 3 equations of equilibrium).

EDSI greater than zero is a necessary but not sufficient condition in order to demonstrate if the structure is stable or not. Additionally, we will check the structure by inspection. A quick look to it shows it is completely linked. For example, add a vertical descending load at point B, and then the structure does not meet the criteria of equilibrium because the balance of moments at point B is different from zero.

However, if the new force applied is a horizontal one at the same point, then it is clear the structure meet the requirements of equilibrium because sum of vertical and horizontal forces and moments are null.

Frame (II)



$\mathrm{EDSI} = \mathrm{R} - \mathrm{EDOF} = 3 - 3 = 0$

$\mathrm{IL} = 2[2.(2-1)] + 2[2.(2-1)] + 2[2.(2-1)]$

$\mathrm{IL} = 24$

$\mathrm{IDOF} = 3.(7-1) = 18$

$\mathrm{IDSI} = \mathrm{IL} - \mathrm{IDOF} = 6$

$\mathrm{DSI} = \mathrm{EDSI} + \mathrm{IDSI} = 6$

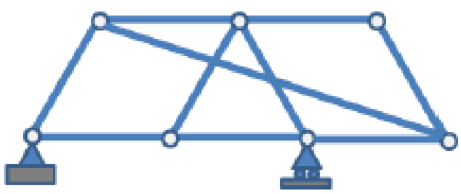
Alternatively:

$\mathbf{DSI} = \mathbf{3m} + \mathbf{r} - \mathbf{3j} = 3.7 + 3 - 6.3 = 6$

Note from the author: prior equation changes from trusses to frames. There are not a lot of exercises solved using this one but as far as I have seen on the internet. If you have any difficulty with it, you might always use the general method using $\mathbf{IL} = \mathbf{3}[(\mathbf{m} - \mathbf{1})]$, where «m» are the numbers of members attached.

In view of the results, the structure is completely linked but its reaction forces cannot be determined using the equations of equilibrium.

Truss (I)



$\mathrm{EDSI} = \mathrm{R} - \mathrm{EDOF} = 3 - 3 = 0$

$\mathrm{IL} = 2[2(2-1)] + 4[2(3-1)] + 2(4-1) = 26$

$\mathrm{IDOF} = 3.(10-1) = 27$

$\mathrm{IDSI} = \mathrm{IL} - \mathrm{IDOF} = -1$

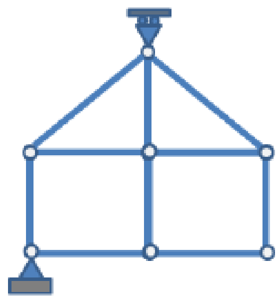
$\mathrm{DSI} = \mathrm{EDSI} + \mathrm{IDSI} = -1$

Alternatively:

$\mathrm{DSI} = \mathrm{r} + \mathrm{m} - 2\mathrm{j} = 3 + 10 - 2.7 = -1$

The DSI of the structure is less than zero therefore it is **partially linked** (mechanism). It also means is unstable due to the existence of a pattern of displacement.

Truss (II)



$\mathrm{EDSI} = \mathrm{R} - \mathrm{EDOF} = 3 - 3 = 0$

$\mathrm{IL} = 2.[2.(2-1)] + 4.[2.(3-1)] + 2(4-1)$

$\mathrm{IL} = 26$

$\mathrm{IDOF} = 3.(10-1) = 27$

$\mathrm{IDSI} = \mathrm{IL} - \mathrm{IDOF} = -1$

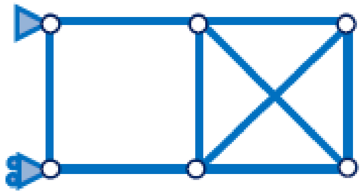
$\mathrm{DSI} = \mathrm{EDSI} + \mathrm{IDSI} = -1$

Alternatively:

$\mathrm{DSI} = \mathrm{r} + \mathrm{m} - 2\mathrm{j} = 3 + 10 - 2.7 = -1$

Then the structure is **partially linked** (mechanism).

Truss (III)



$EDSI = R - EDOF = 3 - 3 = 0$

$IL = 2.[2(2-1) + 2(3-1) + 2(4-1)]$

$IL = 24$

$IDOF = 3.(9-1) = 24$

$IDSI = IL - IDOF = 0$

$DSI = EDSI + IDSI = 0$

Alternatively:

$DSI = r + m - 2j = 3 + 9 - 2.6 = 0$

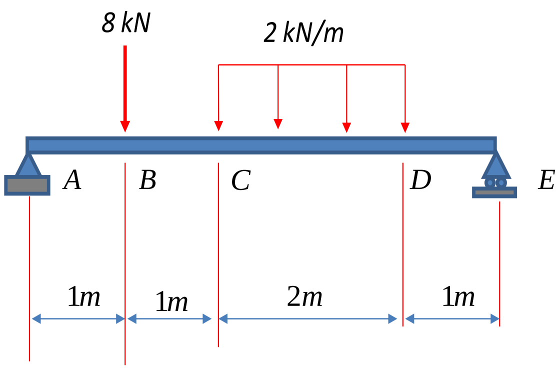
The DSI is equal to zero, **however the structure is partially linked** because it cannot withstand all system of forces because of the limitation of the truss when fulfilling its equation of equilibrium (for more information look for more information of trusses).

Determine for the structure if the figure:

a) An equivalent force system to the one of the figure made up of a point forece at point A and a point force at point E.

b) External degree of static indeterminacy (EDSI).

c) Reaction forces at the supports.



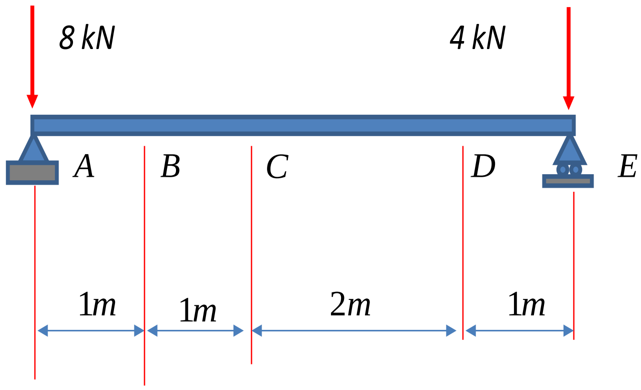
a) <u>Equivalent system of forces.</u>

$\sum F_y = 2.2 + 8 = 12 \text{ kN}$

$\sum M_e = 8.4 + 2.2.2 = 40 \text{ kN.m}$

$5.V_a = 40 \rightarrow V_a = 8 \text{ kN}$

$V_E = 12 - 8 = 4 \text{ kN}$



b) <u>EDSI</u>

$EDSI = 4 - 3 = 1$

The structure is completely linked, because there is no point where all reaction forces would create a null bending moment or similar circumstances that leads to an accelerated structure. Besides, its reaction forces can be calculated through the use of the equations of the equilibrium.

c) <u>Reaction forces</u>

$\sum M_e = 0 \rightarrow 5V_a - 4.8 - 2.2.2 = 0$

$\mathbf{V_A = 8 \text{ kN}}$

$\sum F_y = 0 \rightarrow V_E = 2.2 + 12 - 8$

$\mathbf{V_E = 4 \text{ kN}}$

$$\sum F_x = 0 \rightarrow \mathbf{H_e} = \mathbf{0\ kN}$$

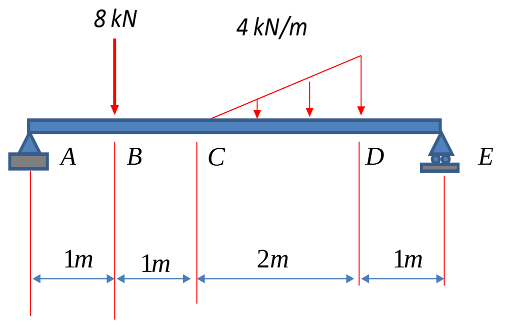
As it is seen, finding the equivalent system is equivalent to determine the value of the reaction forces.

For the structure given in the figure, determine:

a) An equivalent system of forces made up of one on each support.

b) External degree of static indeterminacy (EDSI).

c) Reaction forces.



a) <u>Equivalent force system</u>

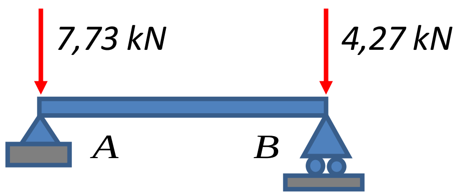
$$\sum F_y = \frac{4.2}{2} + 8 = 12\ \text{kN}$$

$$\sum M_e = 8.4 + 4.\left(1 + \frac{2}{3}\right) = 38{,}67\ \text{kN.m}$$

$$R = (12\ \text{kN};\ 38{,}67\ \text{kN})$$

$$5.V_a = 38{,}67 \rightarrow V_a = 7{,}73\ \text{kN}$$

$$V_E = 12 - 7{,}73 = 4{,}27\ \text{kN}$$



b) <u>EDSI</u>

$$\text{EDSI} = 4 - 3 = 1$$

The structure is completely linked.

c) <u>Reaction forces</u>

$$\sum M_e = 0 \rightarrow 5V_a - 4.8 - \frac{1}{2}4.2.\left(1 + {}^{2}/_{3}\right) = 0$$

$$\mathbf{V_A = 7{,}73\ kN}$$

$$\sum F_y = 0 \rightarrow V_E = 2.2 + 12 - 7{,}73$$

$$\mathbf{V_E = 4{,}27\ kN}$$

$$\sum F_x = 0 \rightarrow \mathbf{H_a} = \mathbf{0\ kN}$$

As it is seen, 1) and 2) lead to the same results.